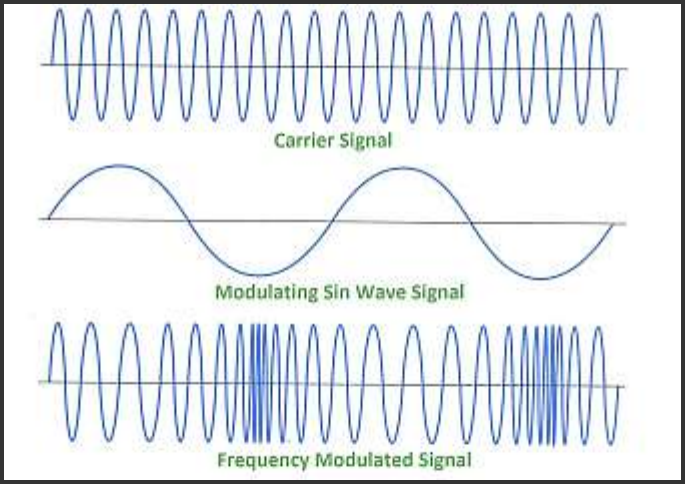
FREQUENCY MODULATION (FM) SYNTHESIS

+ phase distortion (PD)

Frequency modulation (FM)

FM – frequency modulation, used since 1920s to transmit radio waves:

- transmitted signal (modulator) – e.g. radio broadcast
- carrier signal – high frequency sine (e.g. 99.8 MHz)
- amplitude of the transmitted signal modulates instantaneous frequency of the carrier
- modulated signal is transmitted on air
- the received signal is demodulated
- we obtain the original signal



source: http://slidedeck.io/jsantell/dsp-with-web-audio-presentation

FM in sound synthesis

1973 – John Chowning published a paper:

„The Synthesis of Complex Audio Spectra by Means of Frequency Modulation".

- If the two signals have specific frequencies, a harmonic signal is obtained.
- Changes in modulator amplitude modify the timbre.
- Multiple modulations may be performed.
- Easy and cheap method of digital sound synthesis.
- Patented in 1975-1995 by Chowning and Yamaha.

FM in sound synthesis

Let's simplify the problem to two sine oscillators:

- carrier signal (C)

$$x_c(t) = A\sin(\omega t)$$

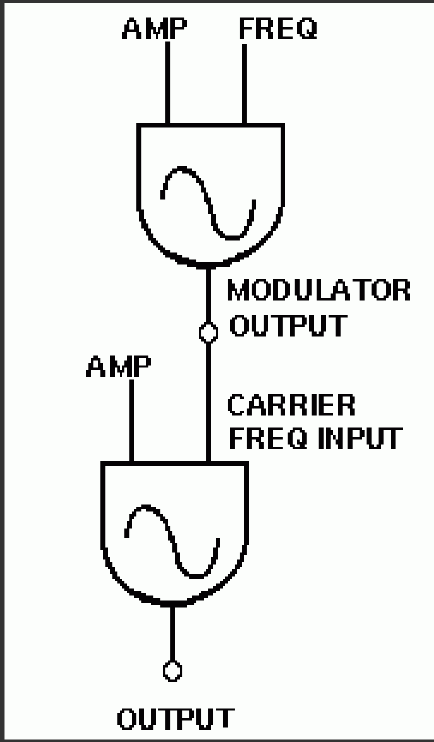
- modulating signal (M)

$$x_m(t) = I\sin(\beta t)$$

The modulator changes (modulates) the instantaneous frequency of the carrier signal:

$$x(t) = A\sin[\omega t + x_m(t)]$$

$$x(t) = A\sin[\omega t + I\sin(\beta t)]$$



Frequency modulation in sound

What effect does FM produce?

- Low modulating frequency (<1 Hz): slow wobbling of the pitch (just like LFO in the subtractive synthesis).
- Modulating frequency in 1 Hz – 20 Hz range: an increasing vibrato effect.
- Frequency above 20 Hz: an inharmonic sound is produced, it sounds very rough.
- In some configurations, e.g. if both frequencies are the same, we get a nice sounding harmonic signal!

Synthetic spectrum components

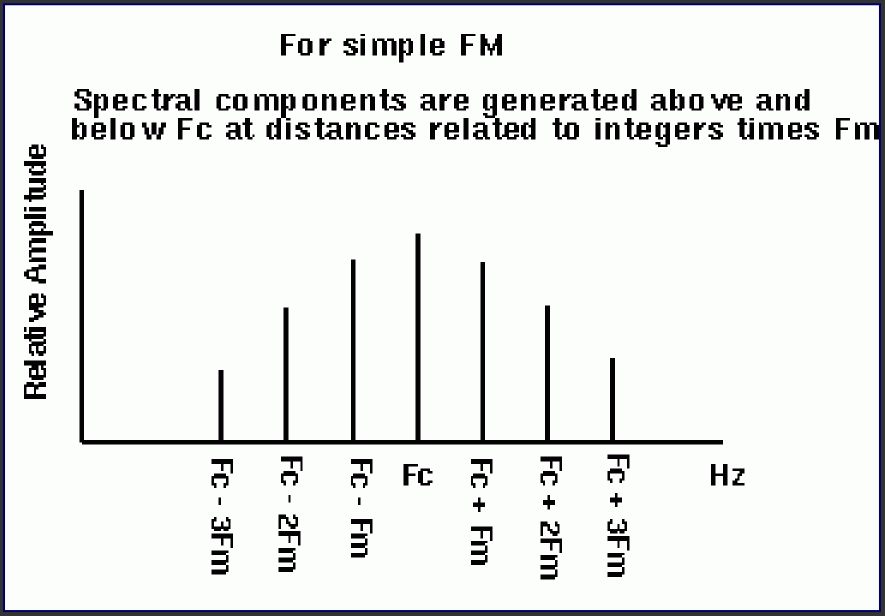
Peaks in the spectrum of a modulated sound:

$$f_c \pm k\, f_m \qquad (k = 0, 1, 2, \ldots)$$

In FM terms: lower and upper band (below and above f_c)

For example, f_c = 500 Hz, f_m = 100 Hz:

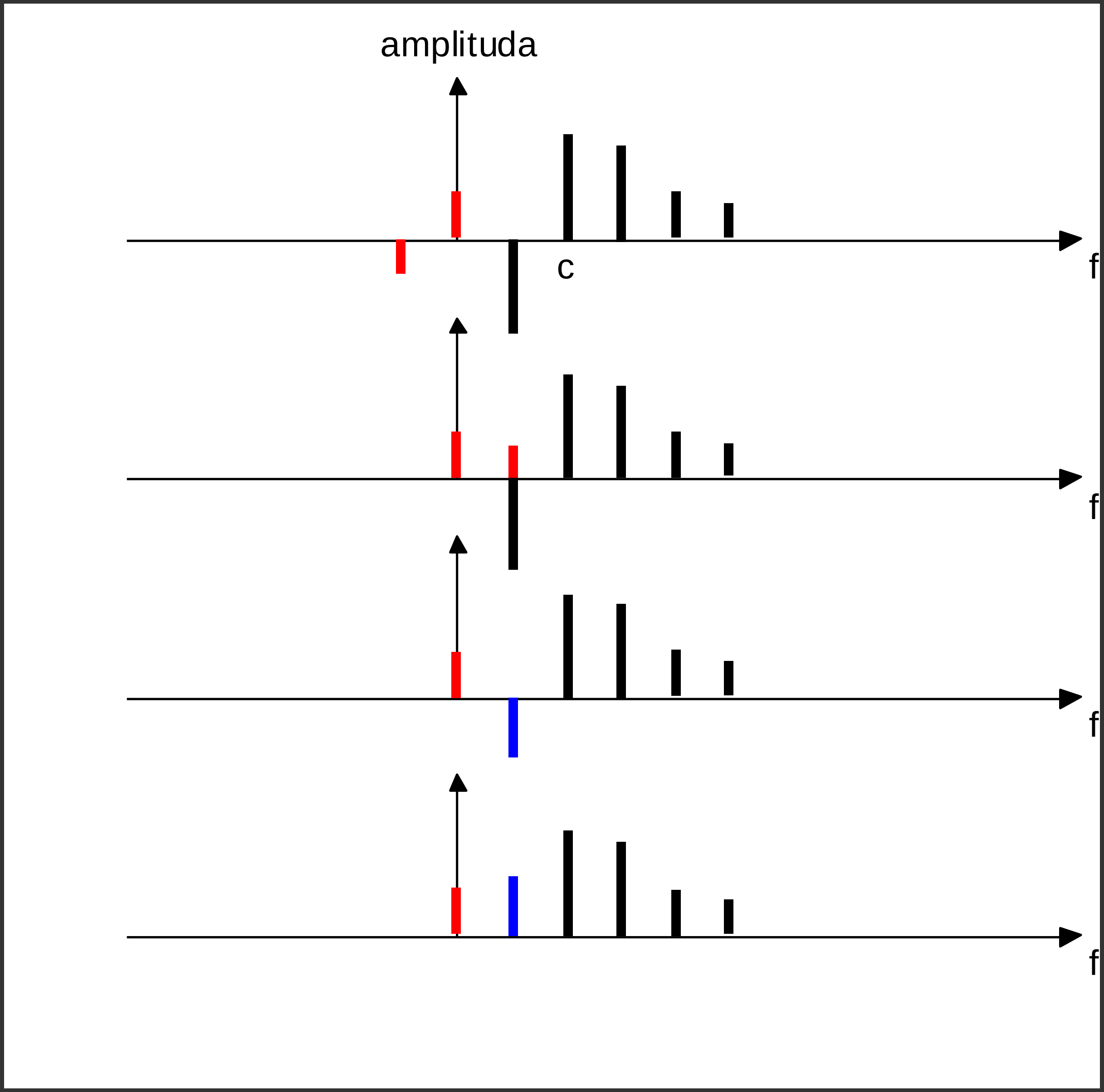
…, 100, 200, 300, 400, 500, 600, 700, 800, 900, …



Reflection of spectral components

- What about components with negative frequencies? For example, for f_c = 400 Hz, f_m = 100 Hz, we obtain: $f_c - 5f_m = 400 - 500 = -100$ Hz
- We know that: $\sin(-x) = -\sin(x)$
- Therefore:
  - a "negative" component is reflected to a positive frequency (an absolute value is taken),
  - phase of the reflected component is inversed,
  - if another component is present at this frequency, amplitudes are summed up (with phase).

Reflection of spectral components



Spectrum with a "negative" component

The component is reflected, its sign changes.

The components are summed, taking their phase into account

Absolute values of the amplitude are taken.

Modulation ratio

Modulation ratio w_m – a ratio of modulating frequency to the carrier frequency.

$$w_m = \frac{f_m}{f_c} = \frac{N_2}{N_1}$$

- In order to obtain a harmonic signal, the modulation ratio has to be expressed as a ratio of integers N_2 and N_1.
- In practice, low integers are used, e.g.: 1:1, 2:1, 3:1, 3:2.

Modulation ratio

Typical values of the modulation ratio
(spectral frequencies are calculated for f_c = 400 Hz):

- 1:1 – all spectral components are present
  400, 800, 1200, 1600, 2000, …
- 2:1 – only even numbered components (k = 0,2,4,...)
  400, 1200, 2000, 2800, …
- 3:1 – every third component is missing
  400, 800, 1600, 2000, 2800, …

Example of an inharmonic spectrum:

- $w_m = \sqrt{2} : 1$

Modulation ratio and fundamental frequency

Warning: this is a common mistake.

Carrier frequency does not have to be equal to the fundamental frequency! The latter is determined by the first peak in the harmonic series.

- f_c = 500 Hz, f_m = 500 Hz $\rightarrow$ f_0 = 500 Hz
  (for modulation ratio 1:1, both frequencies are the same)
- f_c = 500 Hz, f_m = 100 Hz $\rightarrow$ f_0 = 100 Hz
  (the first peak is at 100 Hz: 500 – 4 x 100)
- f_c = 200 Hz, f_m = 300 Hz $\rightarrow$ f_0 = 100 Hz (!!!)
  …, -700, -400, -100, 200, 500, 800, … (reflection:)
  100, 200, 400, 500, 700, 800, …

Modulation index

- Modulation index (I) = modulator amplitude
  (do not confuse with the modulation ratio).
- Determines the modulated frequency range ($\Delta f = I \cdot f_m$).
- Influences the number of important components
  in the spectrum. Larger index – a richer spectrum.
  Carson rule: $B = 2(\Delta f + f_m) = 2\, f_m\, (I + 1)$
- Also influences amplitudes of spectral components
  and therefore, determines the timbre of the sound!
- Practical values: 10 to 100.

Influence of the modulation index

Carrier frequency: 220 Hz, modulation: 440 Hz

Time signals

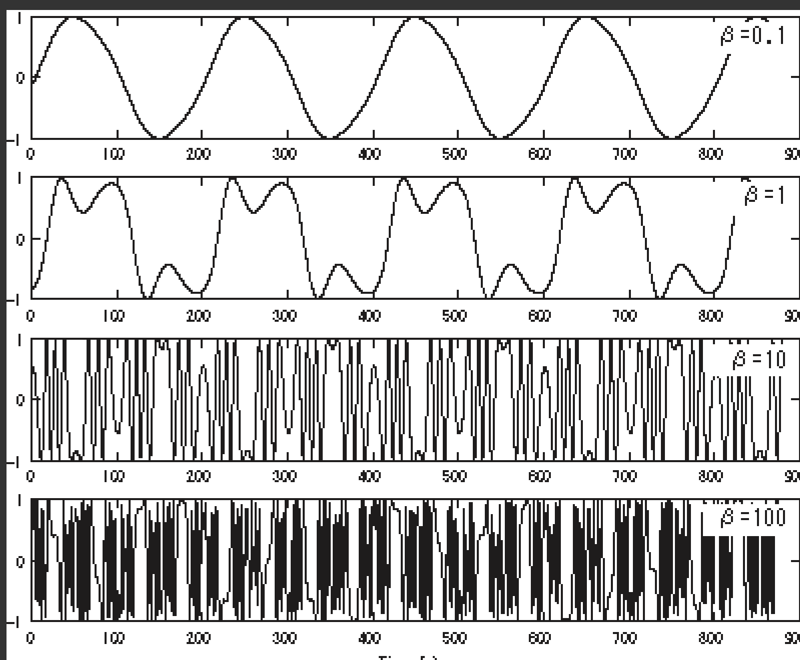
Spectra

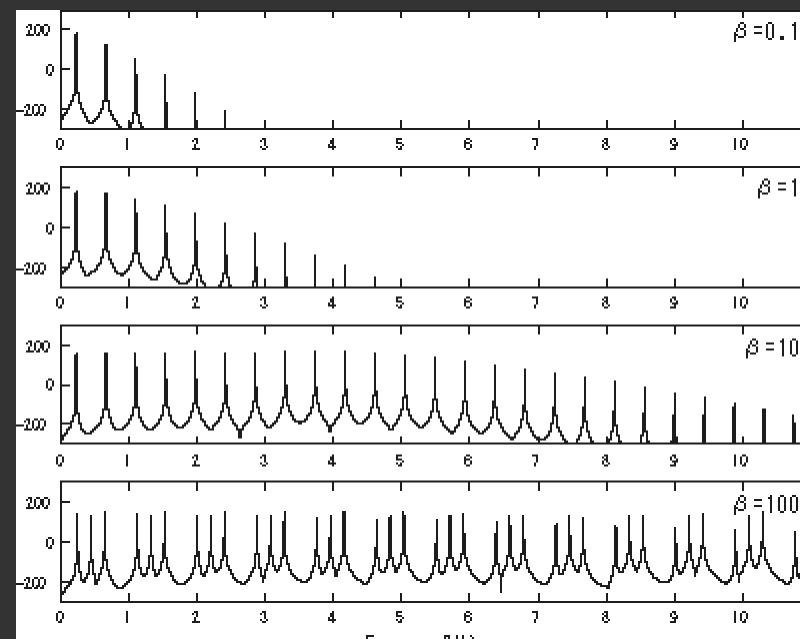
0.1

1

10

100





Amplitude of spectral components

Amplitudes of spectral components are given by:

$$
\begin{aligned}
x(n) = A\{ & J_0(I)\sin(\omega_c nT) \\
& + J_1(I)\cdot[\sin(\omega_c+\omega_m)nT - \sin(\omega_c-\omega_m)\cdot nT] \\
& + J_2(I)\cdot[\sin(\omega_c+2\omega_m)nT + \sin(\omega_c-2\omega_m)\cdot nT] \\
& + J_3(I)\cdot[\sin(\omega_c+3\omega_m)nT - \sin(\omega_c-3\omega_m)\cdot nT] \\
& + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\}
\end{aligned}
$$

Note: odd numbered components in the lower band have inversed phase – negative sign.

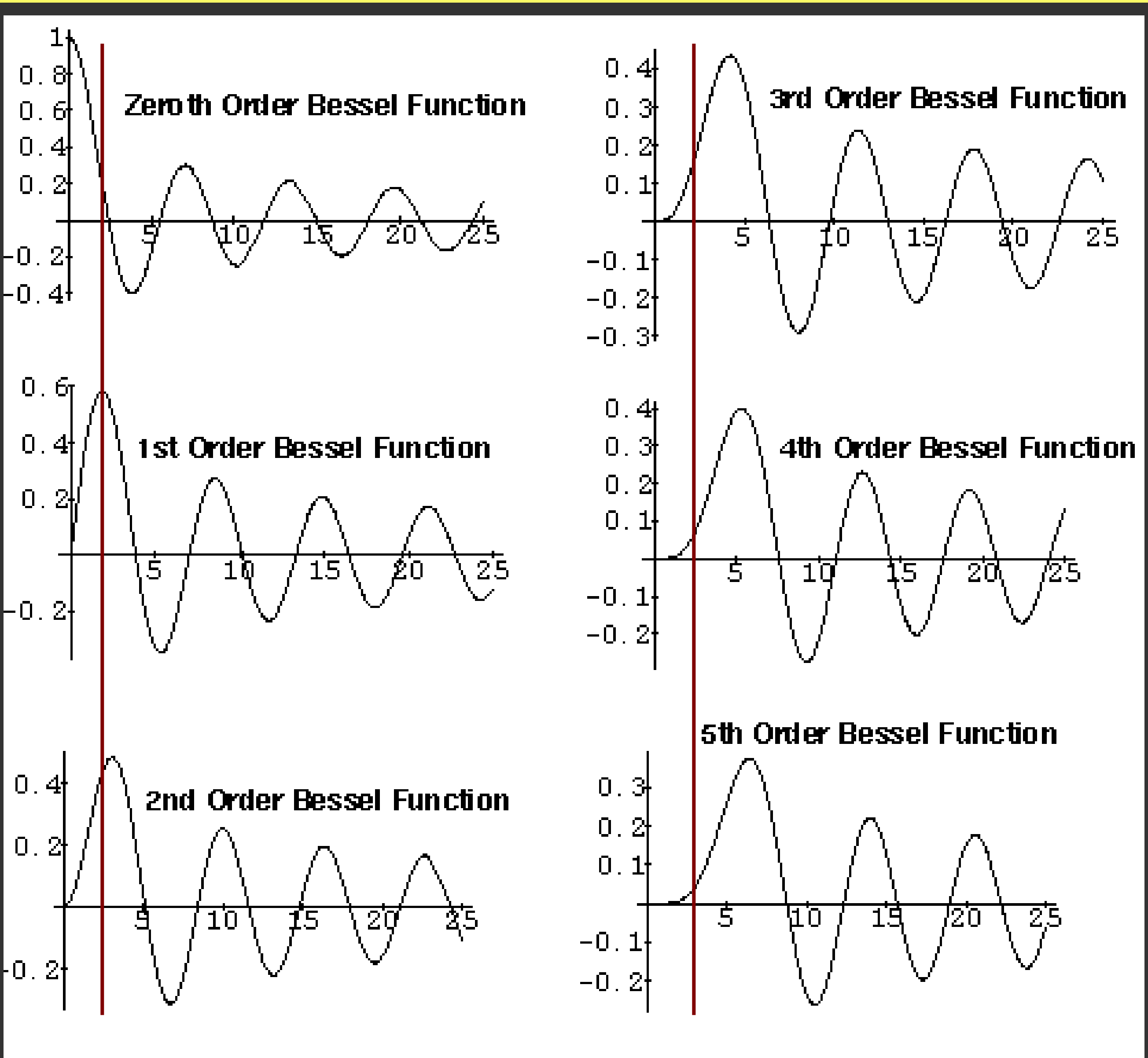
$J_n(I)$: n-th order Bessel functions, argument: modulation idx.

Bessel functions (J)

f_c

$f_c \pm f_m$

$f_c \pm 2f_m$

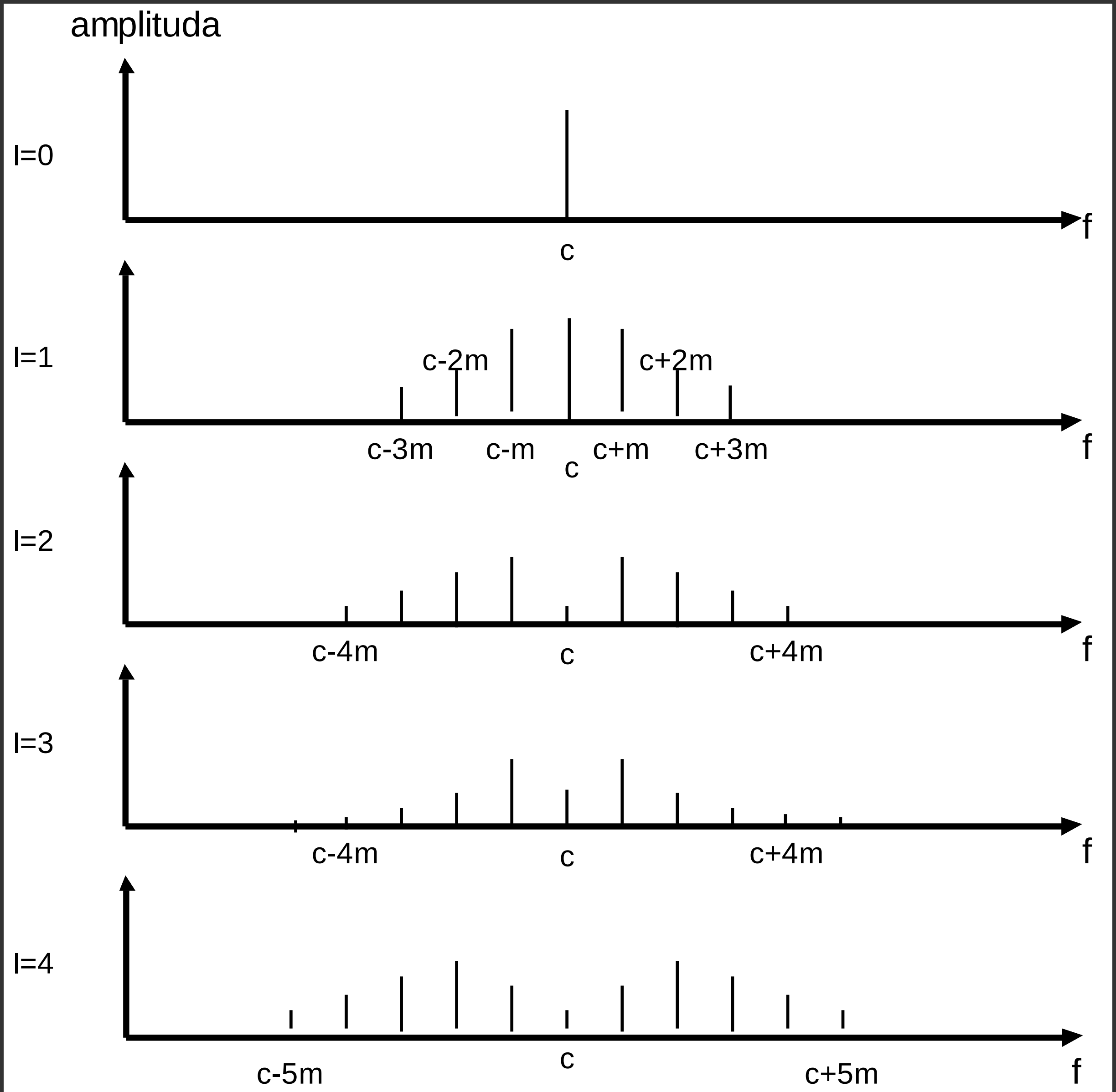


$f_c \pm 3f_m$

$f_c \pm 4f_m$

$f_c \pm 5f_m$

Influence of modulation index on spectrum



Calculating a synthetic spectrum

Parameters: carrier frequency f_c, modulating frequency f_m, modulation index I.

How to compute the spectrum of a FM-modulated signal:

- calculate frequencies of components ($f_c \pm k\, f_m$),
- compute amplitudes of components [$J_k(I)$], remember that some lower band components have negative phase.
- reflect components at negative freqs., invert their phase,
- sum up amplitudes of overlapping components,
- take absolute values of amplitudes.

Note: it is not possible to reverse this process and compute parameters that yield a desired spectrum.

FM synthesis parameters

- Frequencies: carrier (f_m) and modulating (f_m) determine the location of spectral components:
  - they determine if the sound is harmonic,
  - if it is, they determine the sound pitch.
- Modulation index (I) determines amplitudes of spectral components (and, indirectly, the number of components)
  - decide on the sound timbre,
  - modulation index has to be changed during a sound synthesis in order to introduce dynamic timbre changes and make the sound alive.

Operator

Operator is a basic building block of FM synthesis.

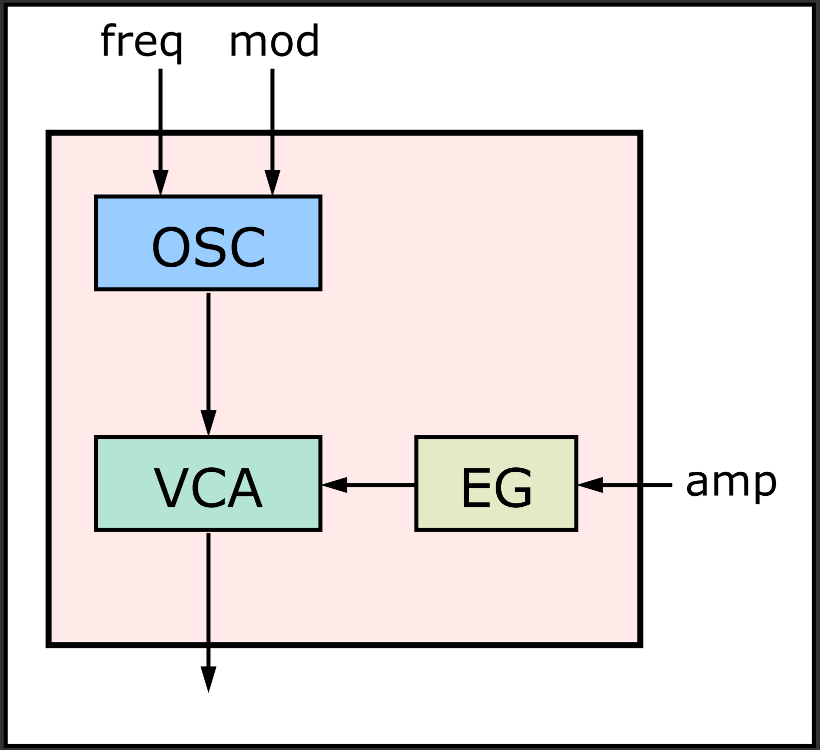
It consists of:

- sine oscillator (OSC)
- amplifier (VCA)
- envelope generator (EG)

freq – fixed frequency

mod – modulating frequency

OSC generates a sine with instantaneous frequency = freq + mod



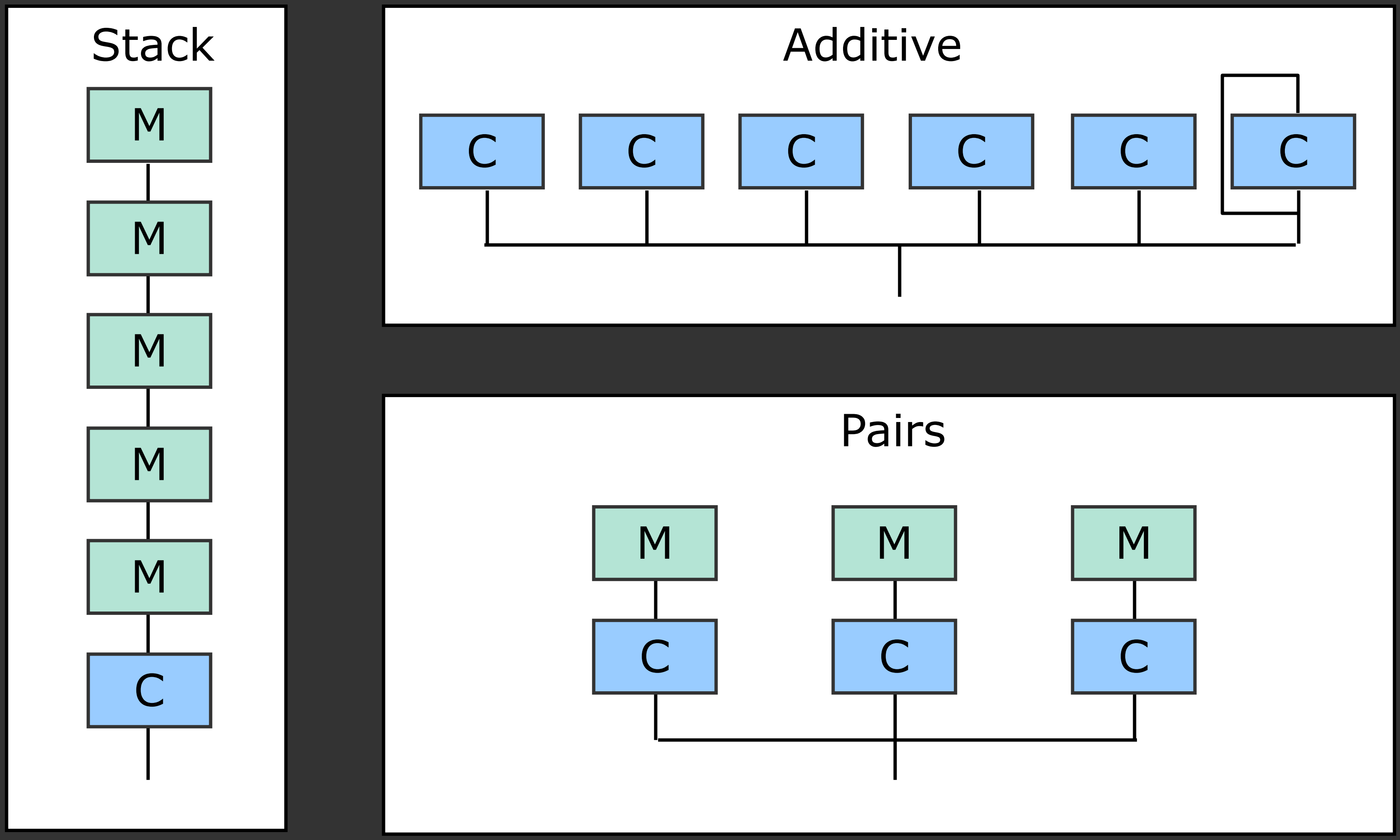
FM algorithm

A connection of two or more operators creates a FM synthesis algorithm.

- Two operators (*Simple FM, 2-op FM*): one carrier and one modulator. The simplest algorithm possible, not sufficient to obtain useful effects.
- In practice, more operators (usually 6) are used, many algorithms are possible.
- The same operators with the same settings, but connected in a different algorithm, produce completely different sound!

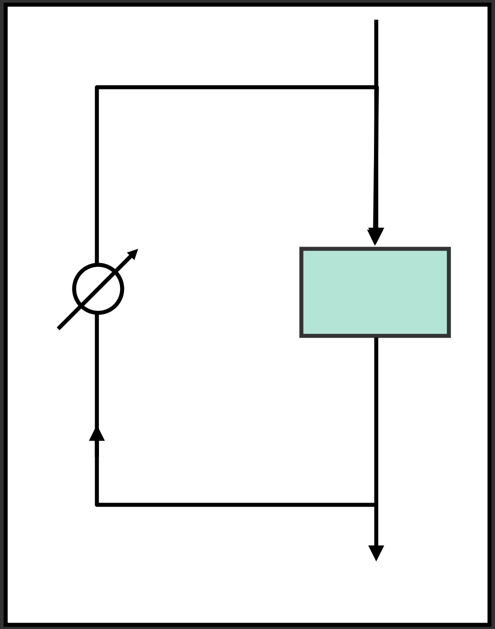
Examples of 6-op algorithms

M – modulator, C – carrier



Feedback

- A modulated signal is fed back to the input and modulated again.
- A gain of the feedback loop is regulated.
- An operator modulates itself!
- Feedback is used to create sounds with reach spectrum (e.g. noise like).



Setting the synthesis parameters

- In an EMI, a pressed key fixes the fundamental frequency.
- For each operator, a frequency multiplier is set.
  The operator generates freq.: fundamental * multiplier.
- Output amplitude in each operator is controlled by EG.
- Amplitudes of carriers (output operators) determine the output level (loudness), EGs control the sound envelope.
- Amplitudes of modulators determine the sound timbre, EGs control modulation index changes.

Setting the synthesis parameters

Sound timbre is controlled with modulation indices
– amplitudes of signals generated by modulating operators.

Modulation index may be controlled by:

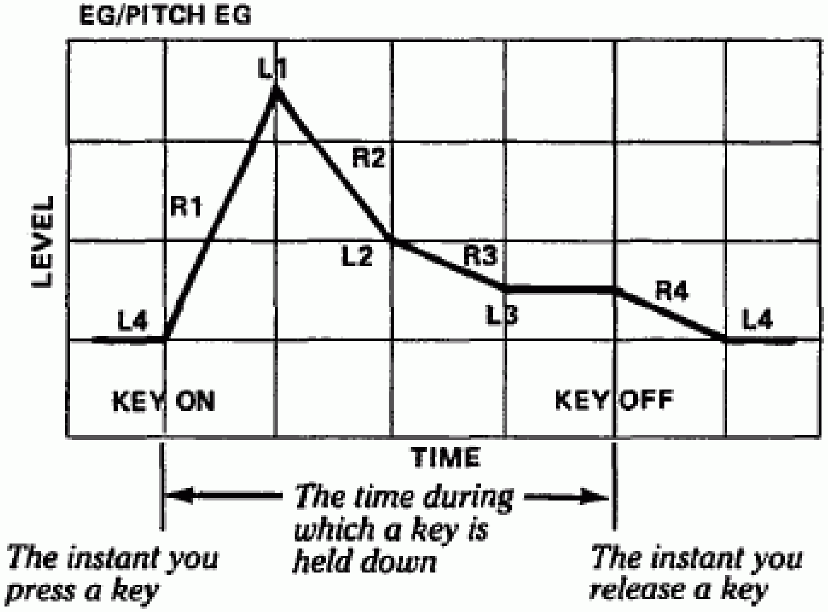
- envelope generators in the modulators – we can modify the timbre, especially in the attack phase,
- LFO blocks – modulation during the sustain phase,
- other controllers, e.g. a modulation wheel.

Yamaha DX7

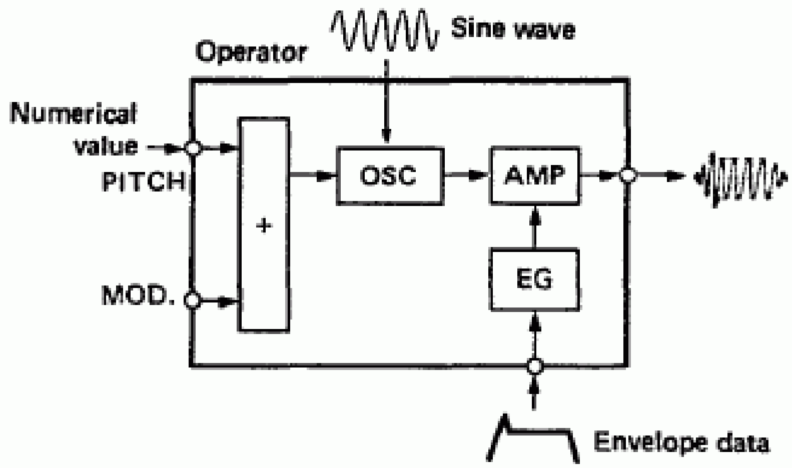
Yamaha DX7 (1983) – the most popular FM synthesizer:

- 6 operators,
- 32 fixed algorithms,
- each operator allows for setting: frequency multiplier, amplitude, envelope and feedback,
- envelope: 4 sections, regulated duration and slope,
- modulators (LFO) and sound effects,
- 16 voice polyphony
- internal and external storage (presets)

Yamaha DX7

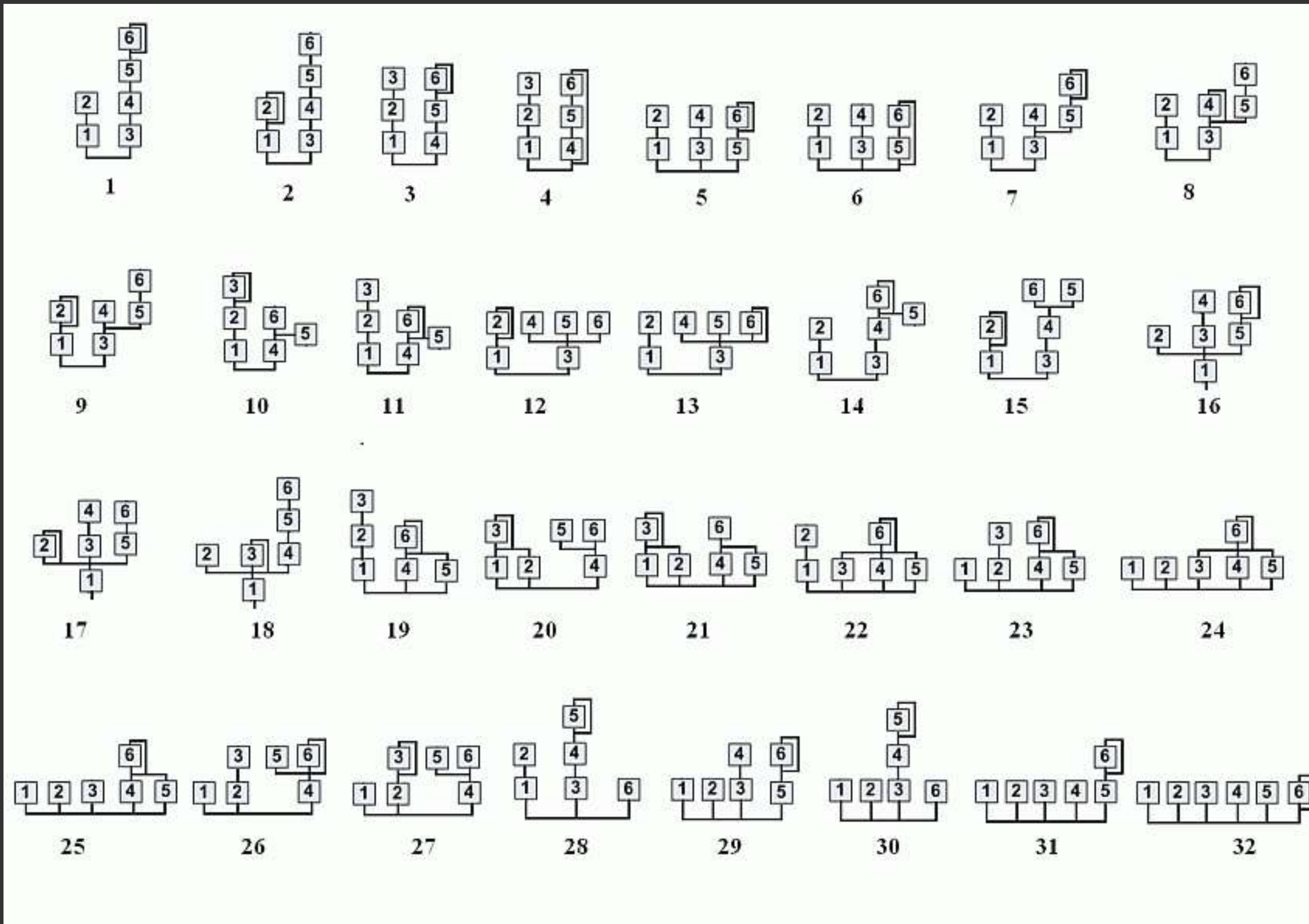


Envelope



Operator

Yamaha DX7 – all 32 algorithms



FM synthesis in PC soundcards

- OPL3 chip by Yamaha, for PC soundcards.
- Used in *Creative Labs SoundBlaster 2/Pro/16* and clones (ca. 1991-94).
- Very simplified FM synthesis: two 2-op and four 4-op algorithms.
- General MIDI compliance: sounds assigned to real instrument names. These sounds were not realistic, which contributed to negative opinions on the FM.
- Replaced by soundcards based on sound samples.

Software FM synthesis

Software FM synthesizers – emulation of hardware synthesizers (*NI FM7* i *FM8*) or custom implementations.

They retain all advantages of the classic FM method.

New functions:

- operators can generate more complex signals than sines, it changes the sound significantly,
- modulation matrices - creating custom algorithms
- additional modules (effects, modulators)

Summary of FM synthesis

Pros:

- interesting and novel sounds (in early 1980s),
- easy and cheap implementation, compared with analogue synthesizers,
- stable pitch,
- many possibilities of sound creation.

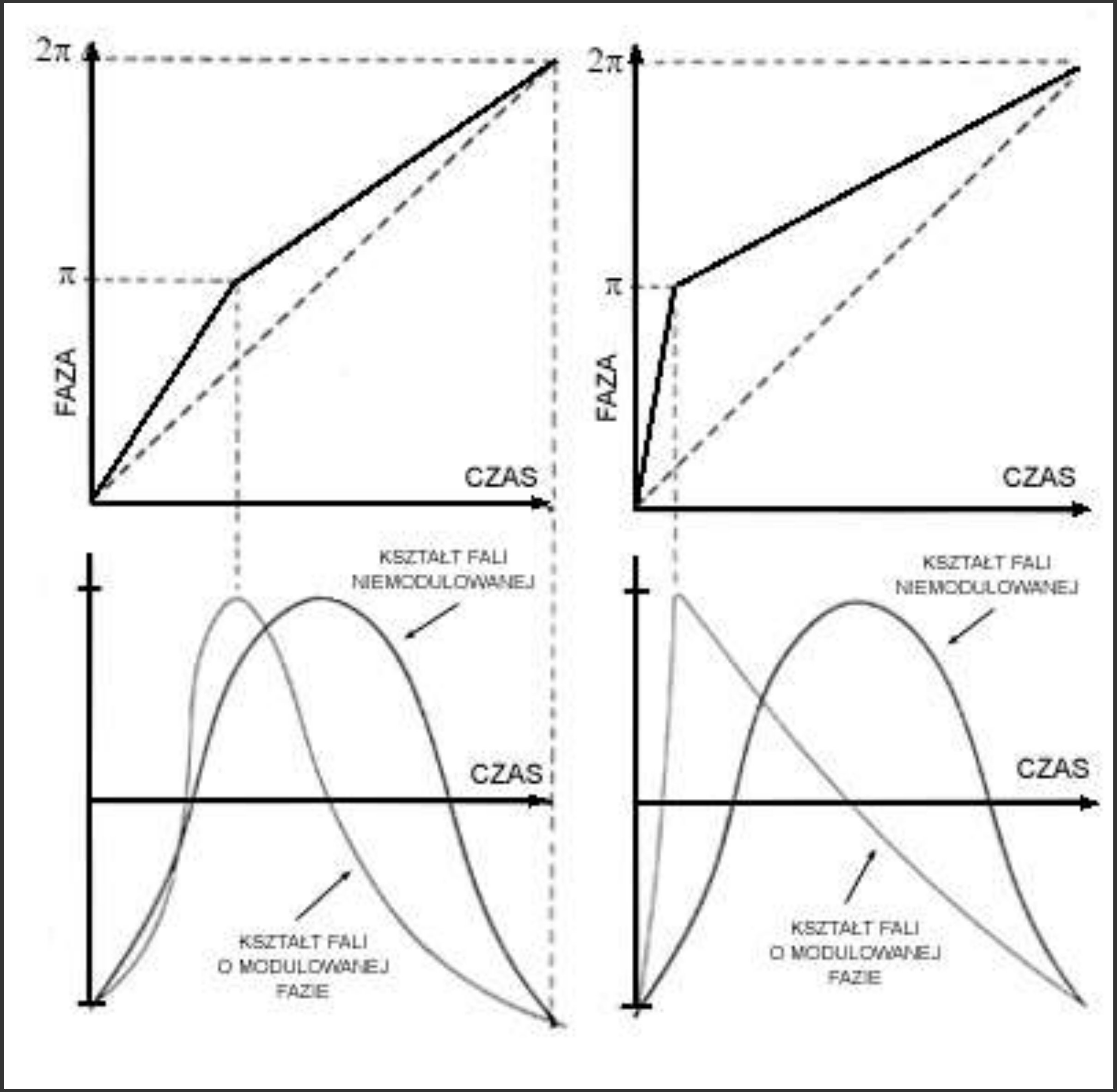
Cons:

- relation between the parameters and the sound is not intuitive,
- for some people, the sound is too artificial ("plastic").

PHASE DISTORTION SYNTHESIS (PD)

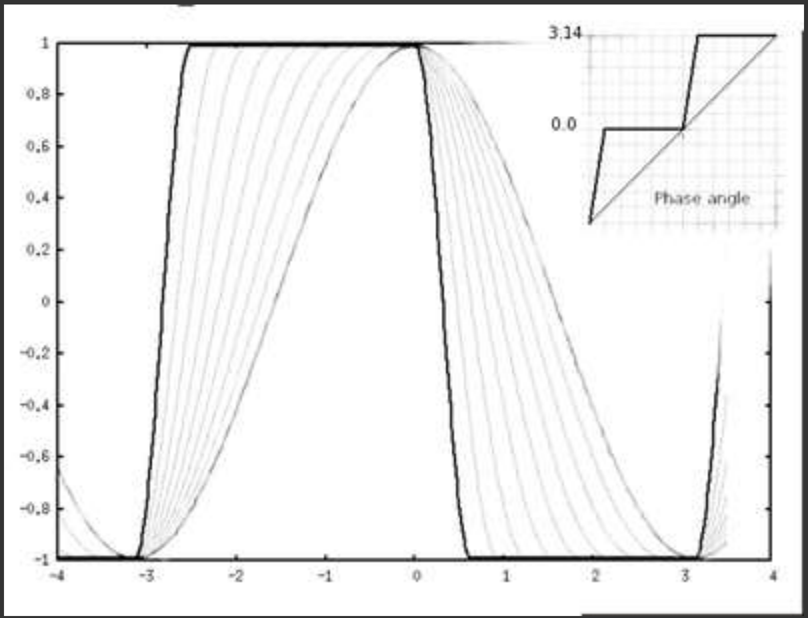
- PD synthesis method was developed by Casio and used in their CZ series instruments (1985-1988).
- Digital, “mathematic” synthesis, similarly to FM.
- Very “synthetic” sounds, almost toy-like.
- The concept: dynamic changes in phase of a sine signal introduce harmonic distortion into the signal and create a sound with a dynamic (changing) timbre.
- The sine signal is read from memory.
- Phase distortion is introduced by varying the speed of reading sine samples from the memory.

An illustration of phase distortion



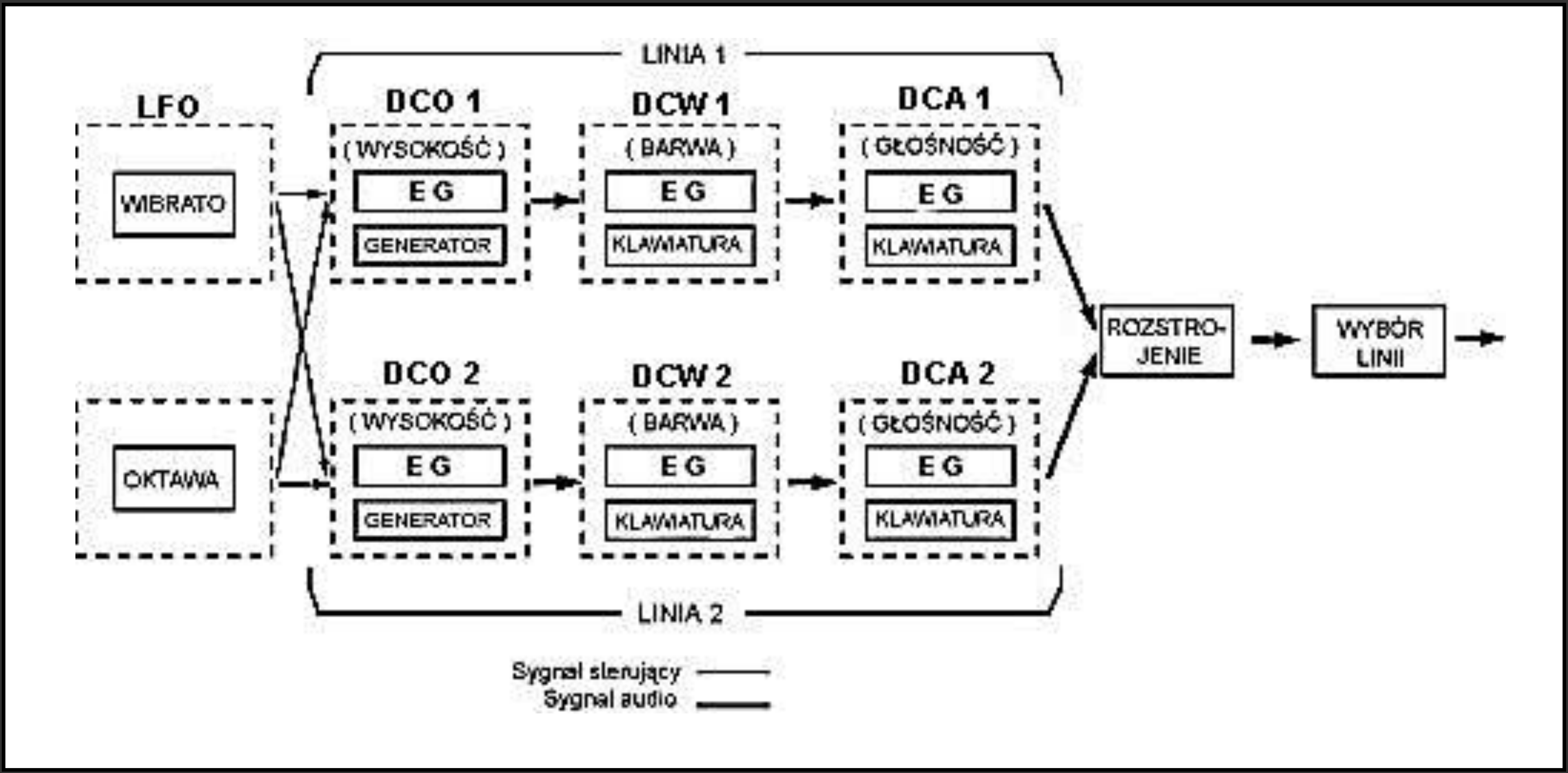
Practical phase distortion

- In practice, timbre changes were achieved by changing the distortion coefficient in 0 to 1 range:
  - 0: pure sine,
  - 1: target signal, e.g. a square wave,
  - between 0 and 1: something in between.
- The distortion coefficient is regulated by envelope generators.
- A "timbre morphing" effect is achieved.



PD instrument

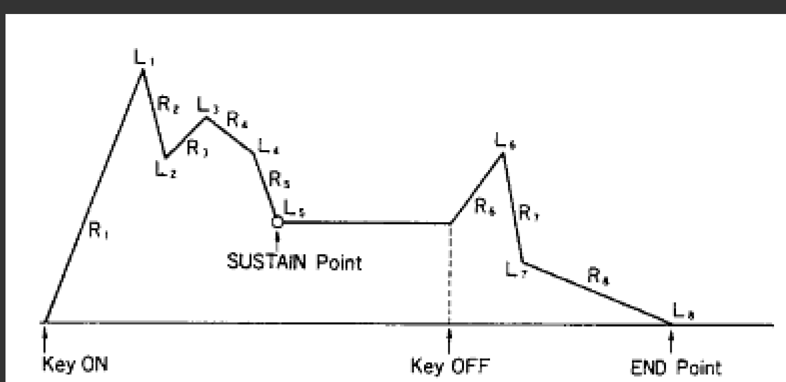
Block diagram of CASIO CZ-01 instrument

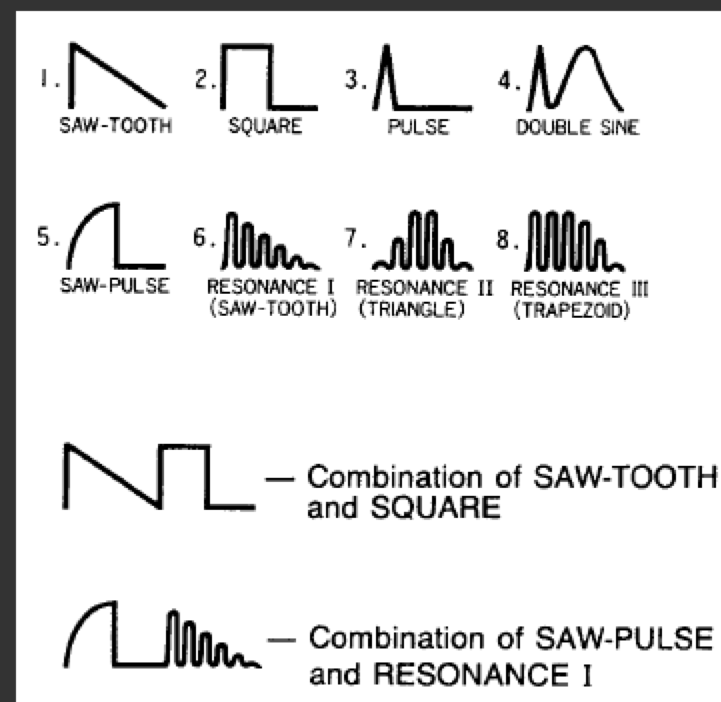


DCW – Digitally Controlled Waveshaper

PD instrument

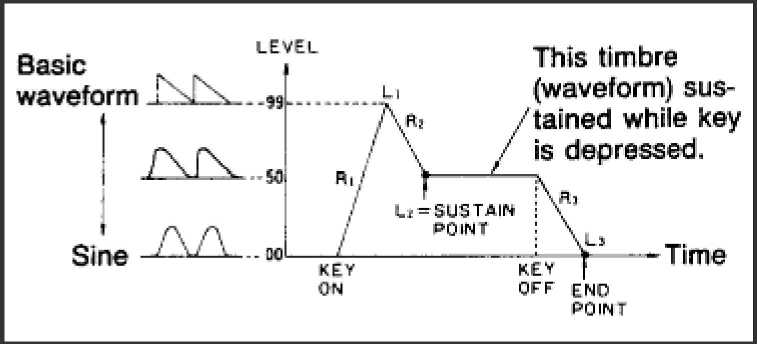
- 8 target wave shapes, stored in memory.
- These shapes can be combined in pairs.
- A total of 33 wave shapes are possible.
- Envelope controls the phase distortion coefficient.

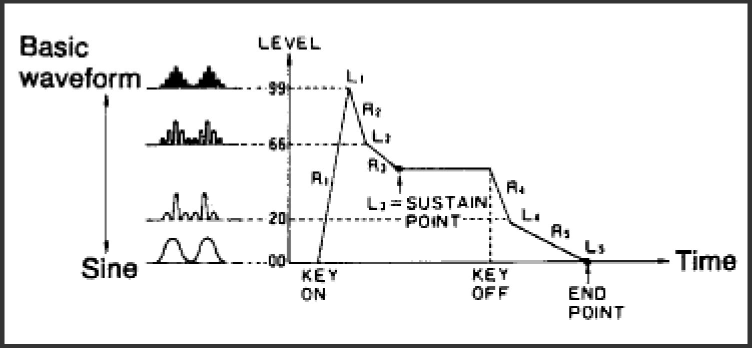




DCW – Digitally controlled waveshaper

- Envelope controls the phase distortion:
  - 0: sine (no distortion),
  - 99: target wave (full distortion).
- *Key follow*: max distortion depends on the key number.
- *Velocity*: max distortion depends on the strength of key press.





Casio CZ instruments

CZ-101 (1985)

CZ-5000 (1985)

CZ-1 (1986)

Summary of PD synthesis

Pros:

- possibility of creation of new, interesting sounds,
- easy to implement and cheap,
- easy to use (small number of parameters).

Cons:

- spectrum cannot be controlled directly,
- FM gives more possibilities of sound creation,
- produces synthetic, “toy like” sounds (but many musicians liked CZ instruments just because of this).

Bibliography

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- Dexed – simple FM synthesizer: https://asb2m10.github.io/dexed/
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